Illustrative Applications

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Gaussian Process Winter School
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About this project

- Joint collaboration

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Ricardo Andrade

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Martin Mubangizi

- Coded on GPy

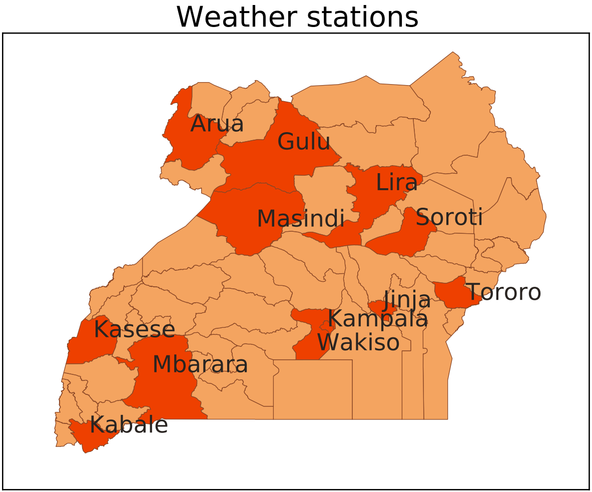
Data

- Health Management Information System
- Weather stations (rain, temperature, humidity)
- Satellite information (vegetation index, rain)
- Geographic data (altitude)
- Sentinel data (incidence)

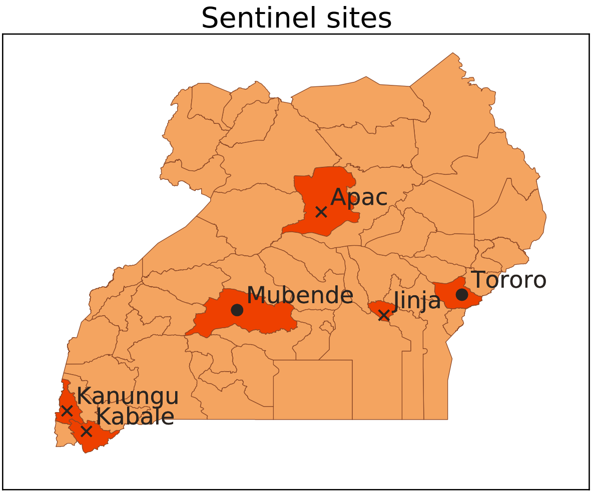
Data features

- Different spatial resolution
- Different temporal resolution
- Noisy records
- Changes in Districts limits along time

Data features



Data features



Multioutput Gaussian Processes (Review)

- D related tasks to be learnt.
- Each one with its corresponding output and inputs: $\{\mathbf{y}_1, \mathbf{X}_1\}, \ldots, \{\mathbf{y}_D, \mathbf{X}_D\}$.
- Each task may have a different number of training examples N_d with $d = 1, \ldots, D$.
- Stack inputs and outputs and treat it as a single output regression problem.

$$\mathbf{X} = \begin{pmatrix} \mathbf{X}_1 \\ \vdots \\ \mathbf{X}_D \end{pmatrix} \qquad \mathbf{y} = \begin{pmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_D \end{pmatrix}$$

Multioutput Gaussian Processes (Review)

- The kernel matrix is partitioned matrix as follows:

$$\mathbf{B} \otimes \mathbf{K} = \begin{pmatrix} B_{1,1} \times \mathbf{K}(\mathbf{X}_1, \mathbf{X}_1) & \dots & B_{1,D} \times \mathbf{K}(\mathbf{X}_1, \mathbf{X}_D) \\ \vdots & \ddots & \vdots \\ B_{D,1} \times \mathbf{K}(\mathbf{X}_D, \mathbf{X}_1) & \dots & B_{D,D} \times \mathbf{K}(\mathbf{X}_D, \mathbf{X}_D) \end{pmatrix}$$

  $\mathbf{K}(\cdot, \cdot)$ is a kernel function,
  $\mathbf{B}$ is known as the coregionalization matrix.

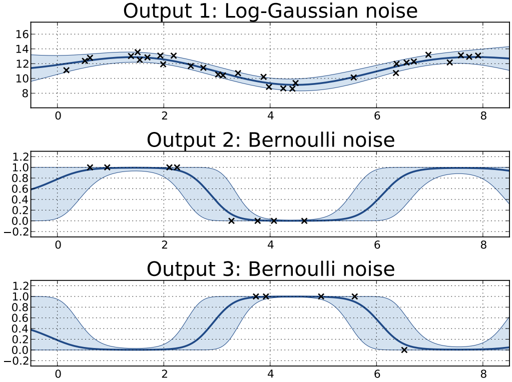
- A more general version:

$$K_{LCM} = \sum_{q=1}^{Q} \mathbf{B}_q \otimes \mathbf{K}_q(\mathbf{X}, \mathbf{X}).$$

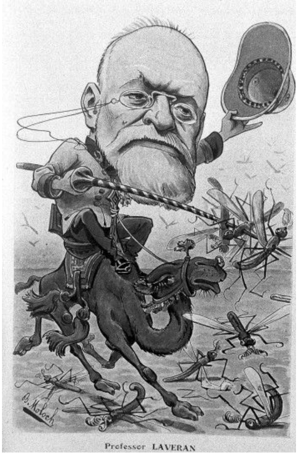
Multioutput Gaussian Processes (Review)

- A different noise term σ_d^2 for each output.

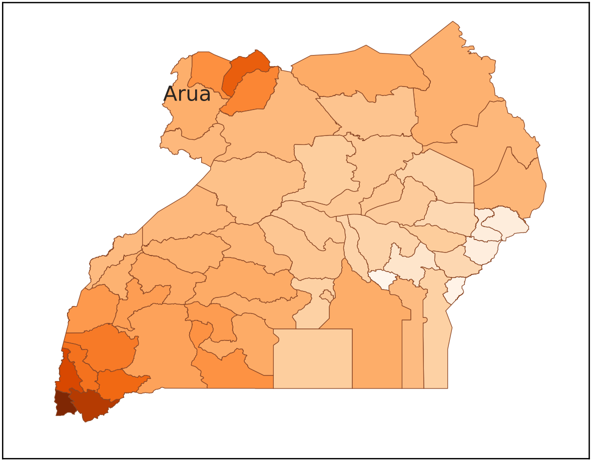
- Even a different noise model for each output.



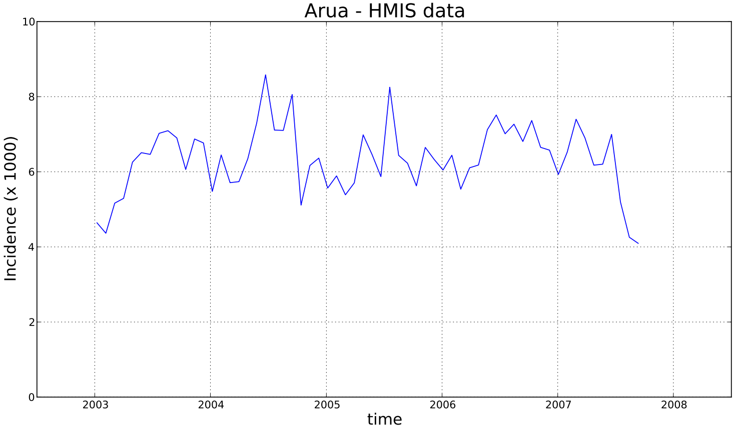
Where to start?



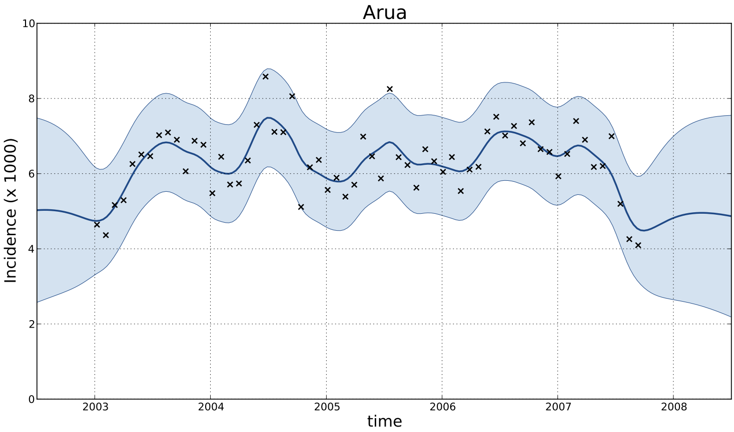
Single output model



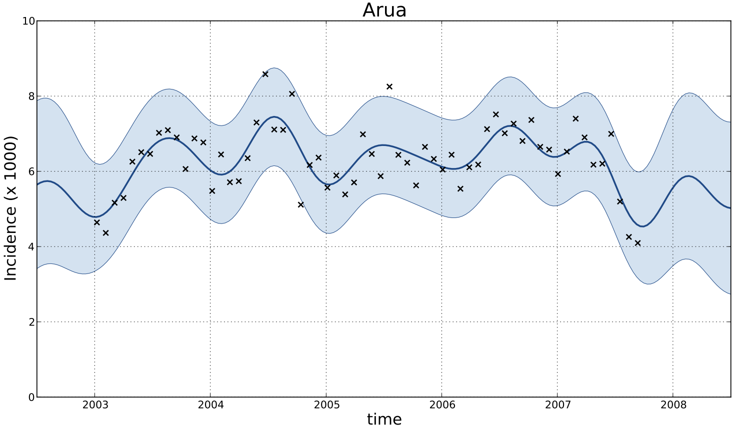
Single output model



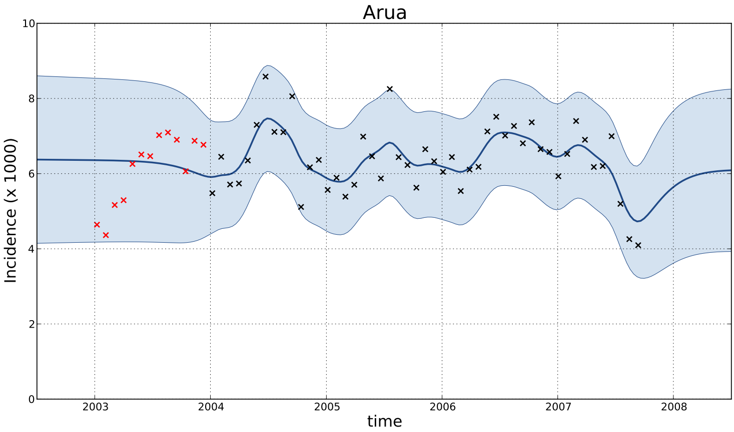
Single output model



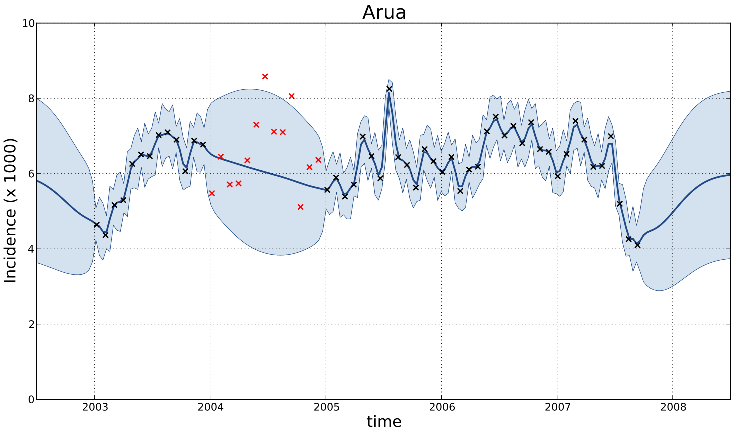
Single output model with a periodic kernel



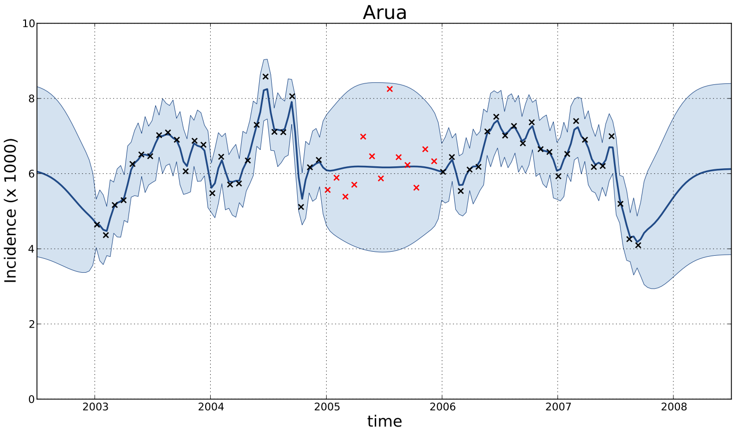
Model without periodic kernel



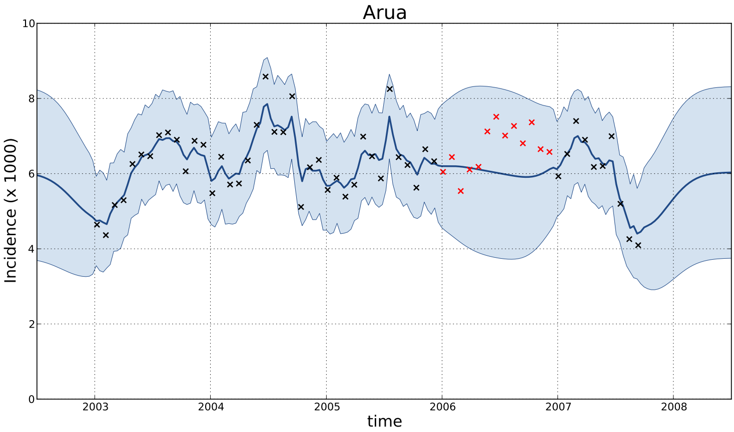
Model without periodic kernel



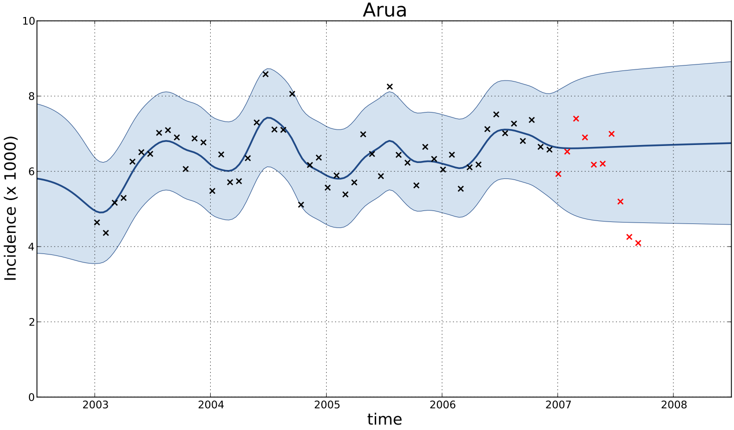
Model without periodic kernel



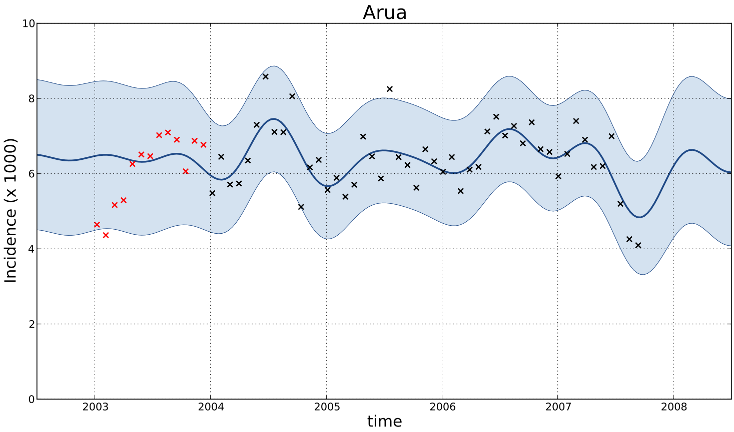
Model without periodic kernel



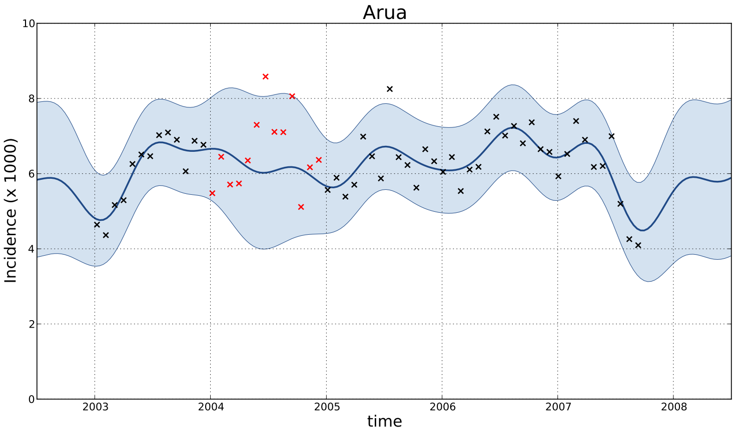
Model without periodic kernel



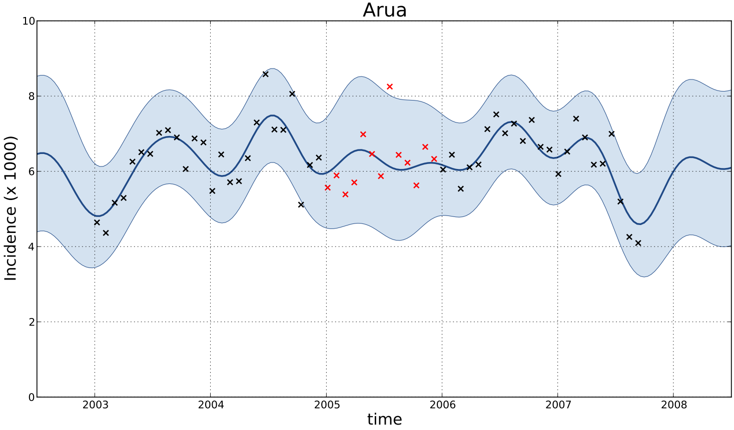
Model with a periodic kernel



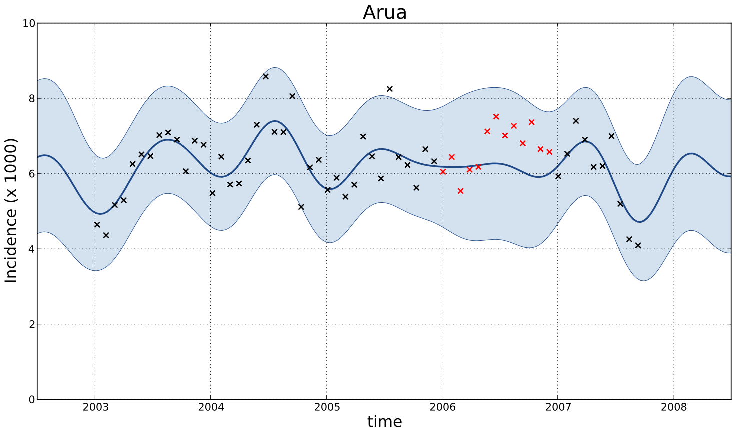
Model with a periodic kernel



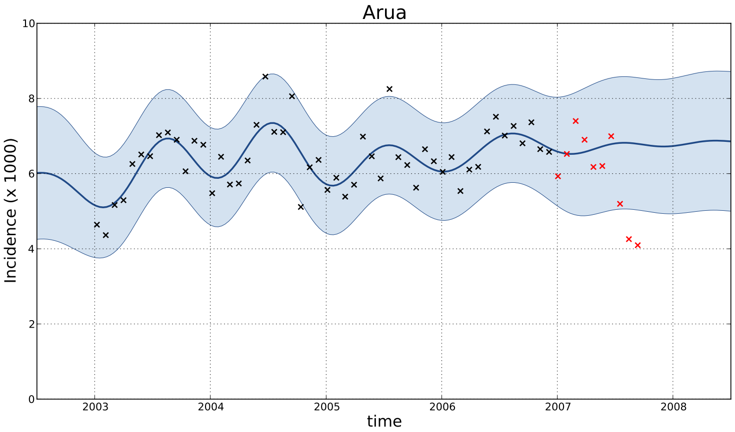
Model with a periodic kernel



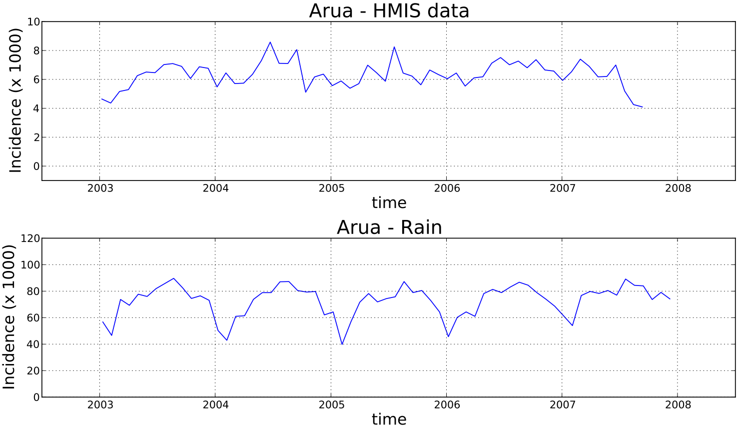
Model with a periodic kernel



Model with a periodic kernel

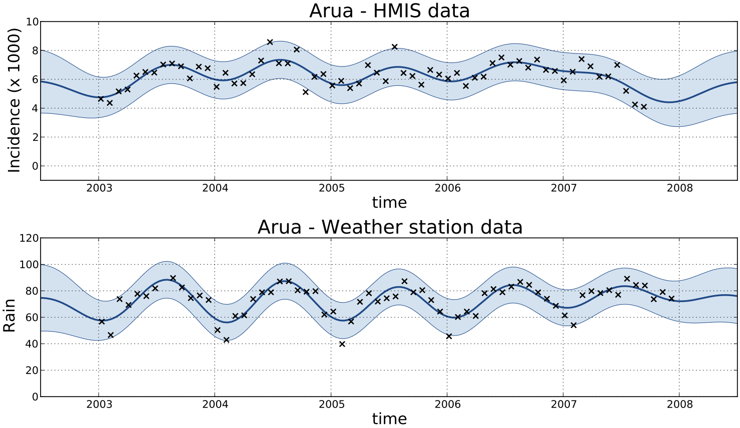


Two outputs



Arua - HMIS data
Incidence (x 1000)
10
8
6
4
2
0
2003
2004
2005
2006
2007
2008
time
Arua - Rain
Incidence (x 1000)
120
100
80
60
40
20
0
2003
2004
2005
2006
2007
2008
time

Two outputs



Arua - HMIS data
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Arua - Weather station data
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2007
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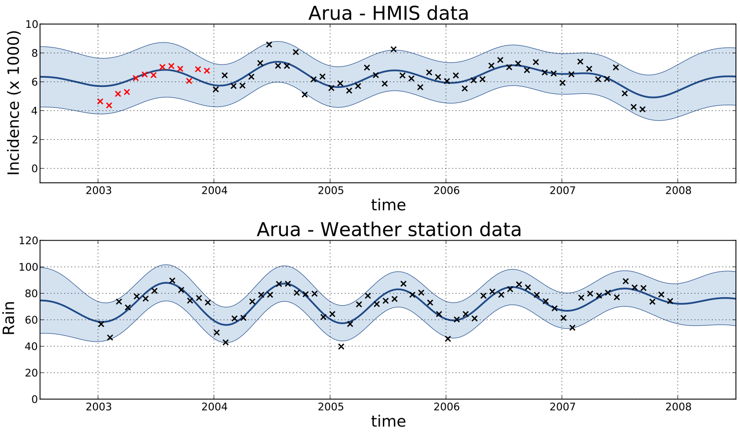
time

Two outputs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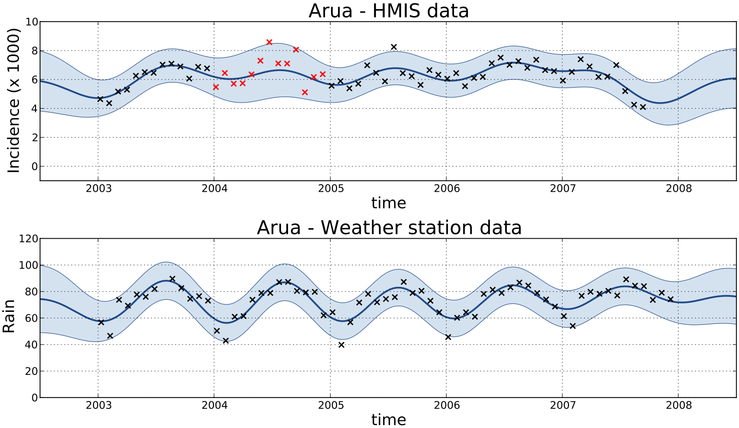

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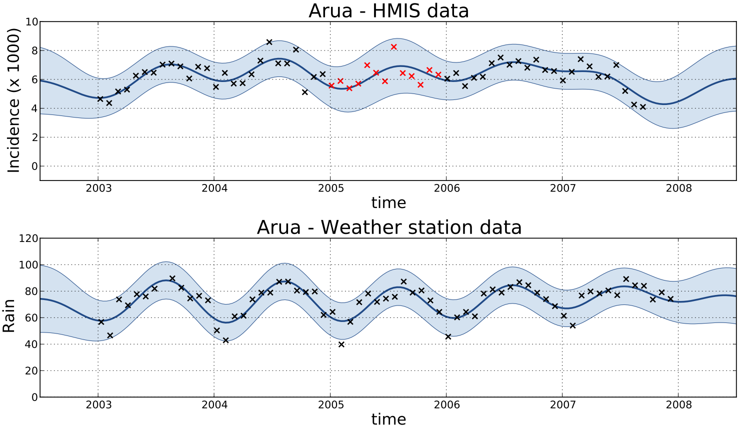
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Incidence (× 1000)
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Arua - Weather station data
Rain
time
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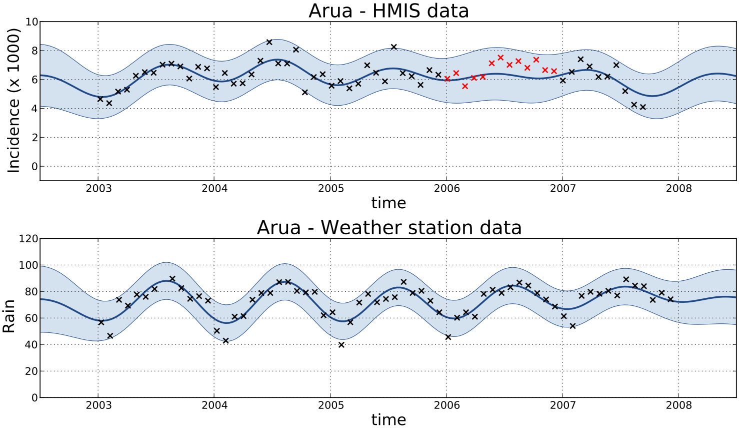
Two outputs



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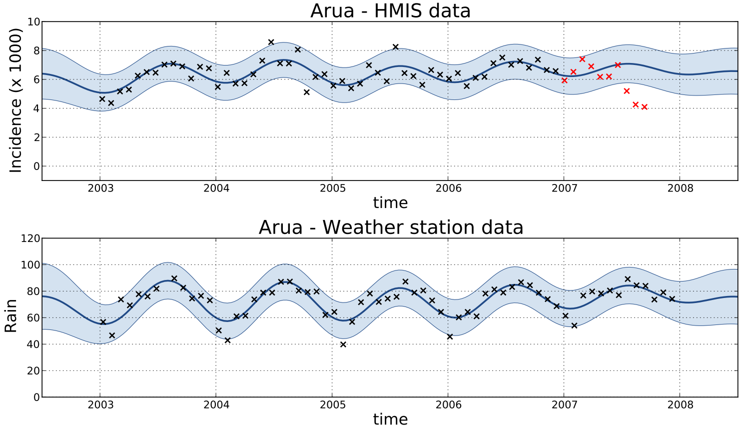
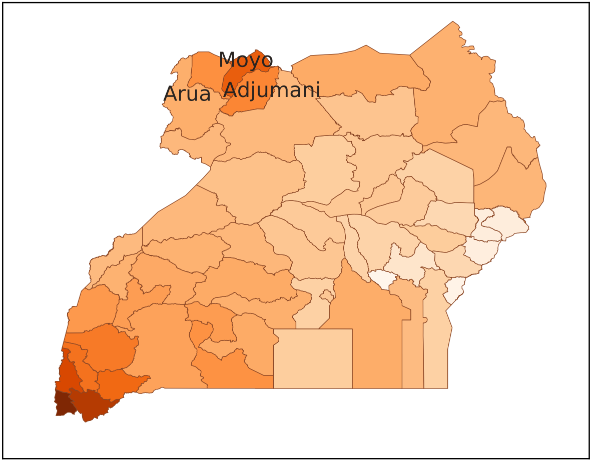
Two outputs


Arua - HMIS data
Incidence (× 1000)
time
Arua - Weather station data
Rain
time

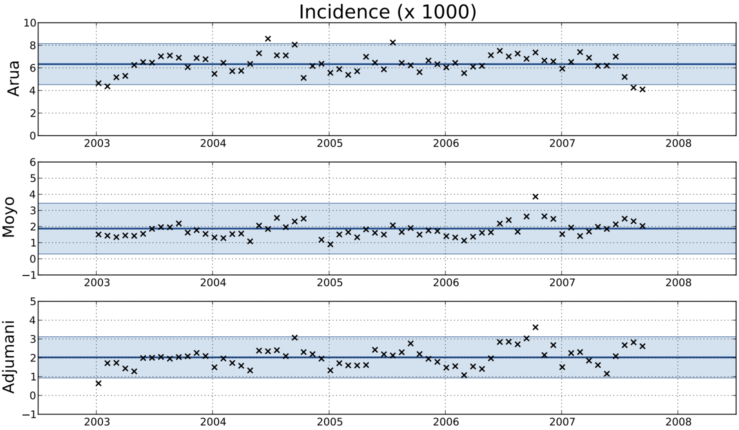
3 districts



Kernel design: bias

- Useful for outputs with different magnitudes
- Otherwise prior standardization is needed

Kernel design: bias



Kernel design: ICM vs LCM

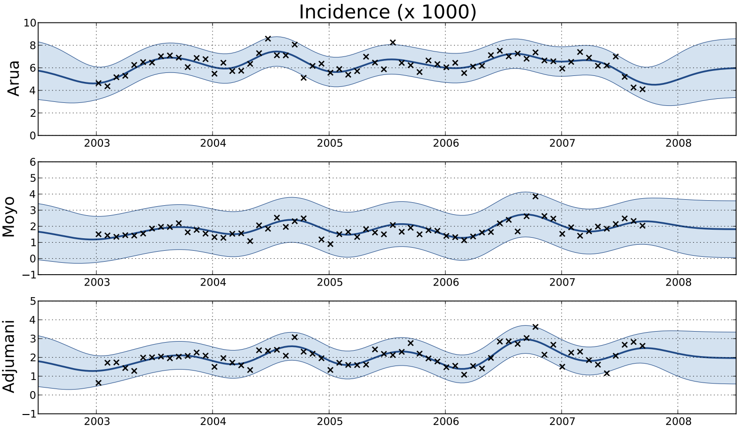
IMC

$$\mathbf{Kernel} = \mathbf{B} \otimes (\mathbf{K}_1 + \mathbf{K}_2)$$

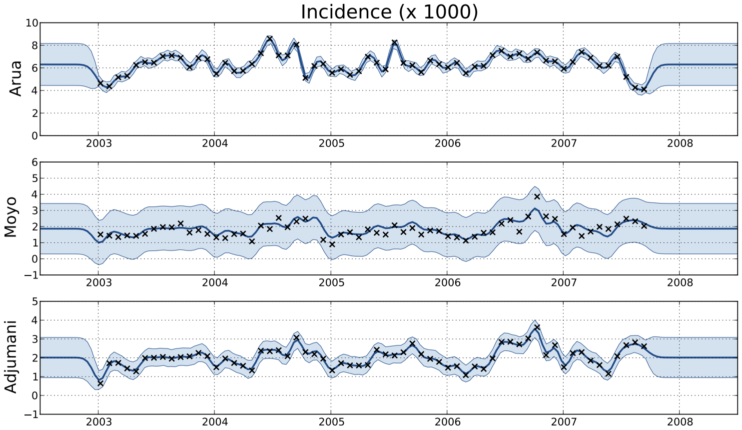
LMC

$$\mathbf{Kernel} = \mathbf{B_1} \otimes \mathbf{K}_1 + \mathbf{B_2} \otimes \mathbf{K}_2$$

Kernel design: ICM



Kernel design: LCM



Kernel design: correlated vs uncorrelated components

Correlated components

$$\mathbf{B} \otimes \mathbf{K} = \begin{pmatrix} B_{1,1} \times \mathbf{K}(\mathbf{X}_1, \mathbf{X}_1) & B_{1,2} \times \mathbf{K}(\mathbf{X}_1, \mathbf{X}_2) & B_{1,3} \times \mathbf{K}(\mathbf{X}_1, \mathbf{X}_3) \\ B_{2,1} \times \mathbf{K}(\mathbf{X}_2, \mathbf{X}_1) & B_{2,2} \times \mathbf{K}(\mathbf{X}_2, \mathbf{X}_2) & B_{2,3} \times \mathbf{K}(\mathbf{X}_2, \mathbf{X}_3) \\ B_{3,1} \times \mathbf{K}(\mathbf{X}_3, \mathbf{X}_1) & B_{3,2} \times \mathbf{K}(\mathbf{X}_3, \mathbf{X}_2) & B_{3,3} \times \mathbf{K}(\mathbf{X}_3, \mathbf{X}_3) \end{pmatrix}$$

Uncorrelated components

$$\mathbf{B} \otimes \mathbf{K} = \begin{pmatrix} B_{1,1} \times \mathbf{K}(\mathbf{X}_1, \mathbf{X}_1) & 0 & 0 \\ 0 & B_{2,2} \times \mathbf{K}(\mathbf{X}_2, \mathbf{X}_2) & 0 \\ 0 & 0 & B_{3,3} \times \mathbf{K}(\mathbf{X}_3, \mathbf{X}_3) \end{pmatrix}$$

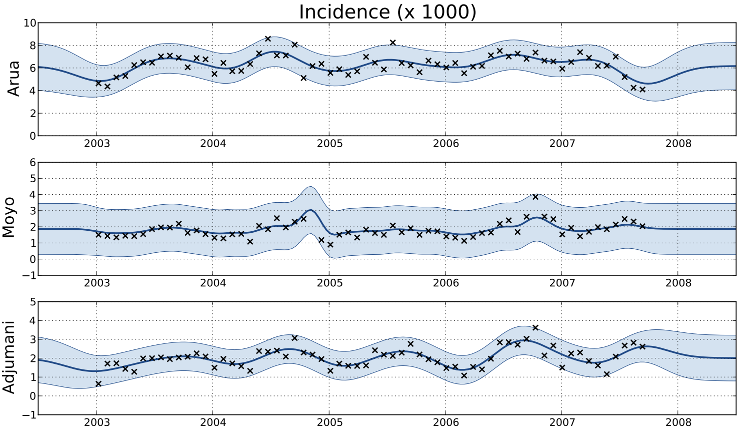
Kernel design: uncorrelated components

Kernel design: private kernel parameters

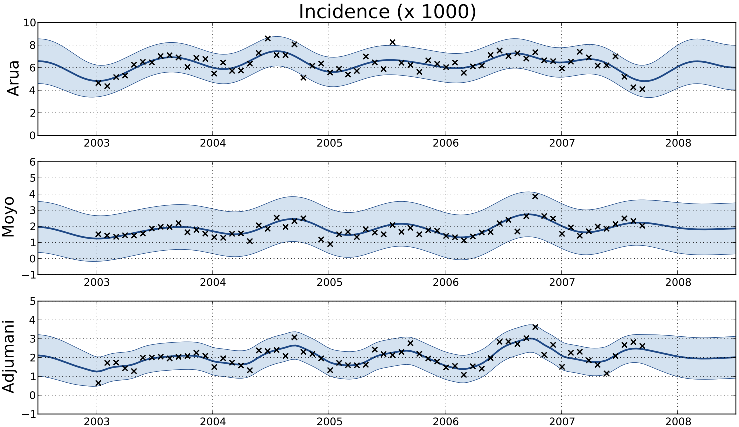
Private kernel parameters

$$\mathbf{B} \otimes \mathbf{K} = \begin{pmatrix} B_{1,1} \times \mathbf{K}_1(\mathbf{X}_1, \mathbf{X}_1) & 0 & 0 \\ 0 & B_{2,2} \times \mathbf{K}_2(\mathbf{X}_2, \mathbf{X}_2) & 0 \\ 0 & 0 & B_{3,3} \times \mathbf{K}_3(\mathbf{X}_3, \mathbf{X}_3) \end{pmatrix}$$

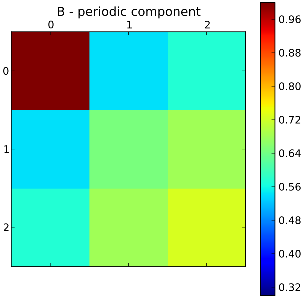
Kernel design: private components

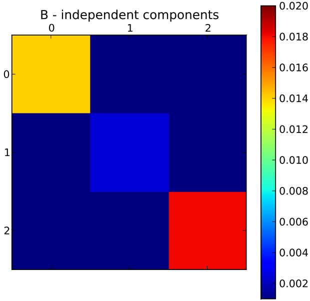


Kernel design: periodic kernel added

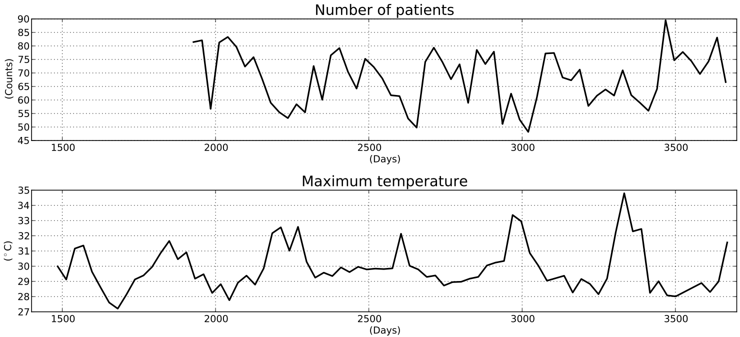


Dominant components





Period extraction: some ideas



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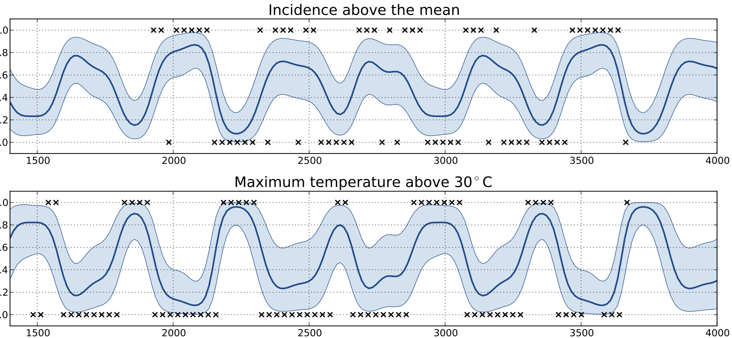
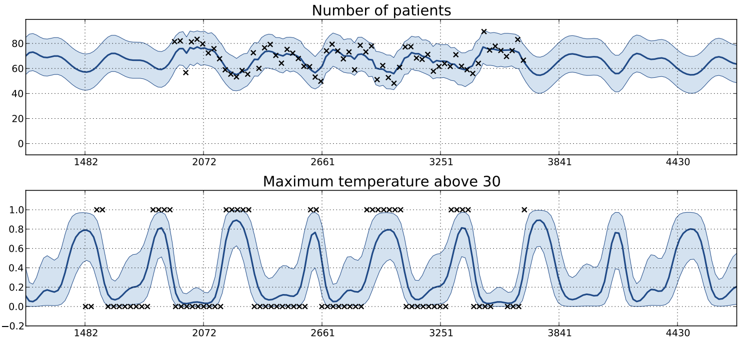


Period extraction: some ideas



Next

- Components dominance and parameters redundancy
- Spatial dimension
- Extend the model to the whole country