Minimum Likelihood Image Feature and Scale Detection Based on the Brownian Image Model

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Abstract

We present a novel approach to image feature and scale detection based on the fractional Brownian image model in which images are realisations of a Gaussian random process on the plane. Image features are points of interest usually sparsely distributed in images. We propose to detect such points and their intrinsic scale by detecting points in scale-space that locally minimises the likelihood under the model.

1 Introduction

Following Marr's paradigm [9] feature detection is the basis on which many vision and image analysis algorithms build upon. The definition of what constitutes an image feature is debatable but there is a common agreement that it is either points or curves of interest where the image intensities have a special geometry. Examples of curve-like features include edges formed by abrupt contrast changes and ridges formed by bar- or valley-like intensity structures. Examples of point features include T-junctions formed by two edges crossing (usually caused by occluding objects), corners formed by two edges meeting at a point, and blobs which are local extrema of the image intensity function. Features have an intrinsic scale that describes their extend in space, e.g. a blob has a certain width. In this paper we use the well-known linear scale space theory [2] for describing the scale of local image structure.

Various methods exist for doing feature detection, but due to space limitations we only provide some references to related work [1, 7, 6, 5, 11, 8, 4].

The method we propose detects features and their intrinsic scale by finding points in scale space with locally minimal probability under a stochastic image model. Hence we propose a definition of features as points occurring

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rarely under a stochastic model of images. We use the covariance structure of the fractional Brownian image model leading to a Gaussian process model of images. Notice that contrary to other methods for feature detection we do not include an explicit model of the features we want to detect.

2 Theoretical background

In order to describe local image geometry we use the scale space jet representation [2, 3]. The scale space $L : \mathbb{R}^2 \times \mathbb{R}_+ \to \mathbb{R}$ of an image $f : \mathbb{R}^2 \to \mathbb{R}$ is given by convolution with a Gaussian blurring kernel

$$L(x, y; \sigma) = (G_\sigma * f)(x, y) \tag{1}$$

where σ is the measurement scale and

$$G_\sigma(x, y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right) . \tag{2}$$

So-called scale normalised scale space derivatives may be computed by

$$L_{x^n y^m}(x, y; \sigma) = \sigma^{n+m} \left(\frac{\partial^{n+m} G_\sigma}{\partial x^n \partial y^m} * f\right)(x, y) . \tag{3}$$

The scale space k-jet at an image point is the vector of partial scale space derivatives up to order k of the image intensity function at that point,

$$j_\sigma(x, y) = (L_x, L_y, \ldots, L_{x^n y^m})^T \tag{4}$$

where $n+m = k$. We disregard the zeroth order term since it does not carry any relevant local geometric information.

In order to derive our Gaussian stochastic image model we start by considering the fractional Brownian image model [10]. The covariance matrix of scale normalised jets of fractional Brownian images can be computed analytically (see [10]). In the following $n_1 m_1$ and $n_2 m_2$ indicates the derivation order for both filters. The covariance is

$$\Sigma_{nm}^{\{\sigma,\alpha\}} = (-1)^{\frac{n+m}{2}+n_2+m_2} \frac{\beta(n-1)!!(m-1)!!}{4\pi\sigma^{2-\alpha}(n+m)!!} \Gamma\left(\frac{n+m-\alpha}{2} + 1\right) \tag{5}$$

whenever both $n = n_1 + n_2$ and $m = m_1 + m_2$ are even integers, otherwise $\Sigma_{nm}^{\{\sigma,\alpha\}} = 0$. Double factorial is defined as $n!! = n(n-2)(n-4)\cdots$. The α parameter dictates the spatial correlation structure of the model and valid choices are $1 < \alpha < 3$. Choosing $\alpha = 2$ leads to the scale invariant Brownian image model which is a Gaussian process with i.i.d. Gaussian increments $f(x_1, y_1) - f(x_2, y_2)$. The β constant describes the variance of the intensities and is not of interest here.

Fractional Brownian images are not in general Gaussian processes, but we are only interested in the covariance of jets of this model and since we discard the zeroth order term in the jet we effectively assume that images are zero mean. Using only this information leads to a Gaussian assumption on images and the probability density on jets

$$p_\alpha(j_\sigma(x,y)) = \frac{1}{Z} \exp\left(-\frac{1}{2} j_\sigma^T(x,y)(\Sigma^{\{\sigma,\alpha\}})^{-1} j_\sigma(x,y)\right) \tag{6}$$

where Z is a normalisation constant and the covariance $\Sigma^{\{\sigma,\alpha\}}$ is given by (5).

3 Feature and Scale Detection

We define features points as points of low probability of occurrence under the fractional Brownian image model, i.e. minimal likelihood, and we propose to detect the intrinsic scale in a similar way.

Our method can be summarised as follows: For a particular choice of α find points in scale space which locally minimise the likelihood under the model given by (6),

$$(\hat{x}, \hat{y}, \hat{\sigma}) = \arg \min_{(x,y,\sigma)} p_\alpha(j_\sigma(x,y)) \ . \tag{7}$$

Minimising the likelihood is similar to the well-known feature and scale detection method proposed by Lindeberg [7]. Lindeberg maximises so-called measures of feature strength which are polynomials of image derivatives. In our setting, $-\frac{1}{2} j_\sigma^T(x,y)(\Sigma^{\{\sigma,\alpha\}})^{-1} j_\sigma(x,y)$ correspond to such a feature strength, hence maximising this measure is equivalent to minimising the likelihood.

4 Preliminary Results

As a preliminary demonstration of our method we include results of detecting features on a synthetic double blob image. We applied our method to this image using the 4th order jet and the Brownian image model $\alpha = 2$. The results are found in fig. 1. Our method detects the two blobs position and scale correctly. Besides this it also detects the high scale edge surrounding the blob (the two off centre dots at scale $\sigma = 30$).

5 Conclusion

We present a novel method for detecting image features and their intrinsic scales. This is done by finding local minima in scale space of the likelihood

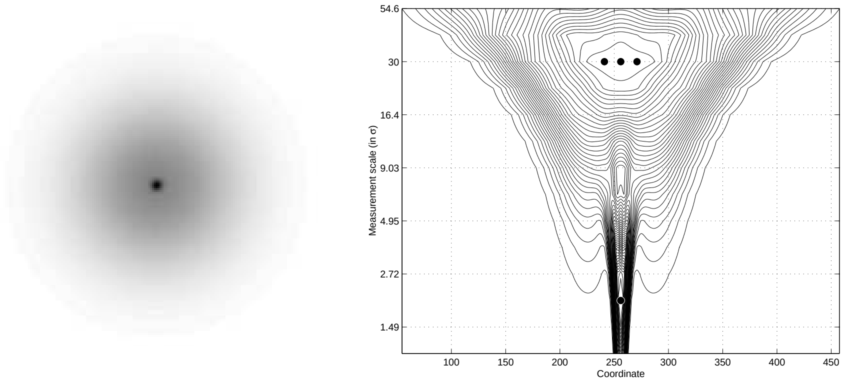

Figure 1: (Left) Synthetic image of two blobs with scales $\sigma = 2$ and $\sigma = 30$. (Right) Contour plot of the probability of the middle cross section of the left image across scale. The four dots represents local minima.

of the image under the fractional Brownian image model. We presented promising but preliminary results on a synthetic image. Obviously the next step would be a thorough investigation on real natural images. There seems to be a connection between the choice of α, order of jet, and which features and scale we detect and we would like to investigate this further.

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