

# Calibration of stochastic models using history matching and emulation.

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GPUQSS 2016, Sheffield, 15 September 2016

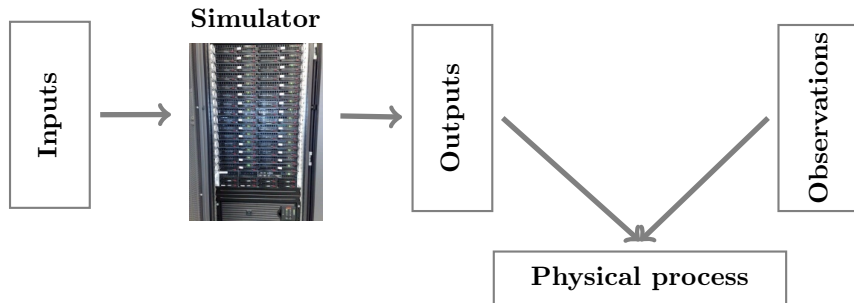
*Joint work with: I. Vernon, N. McCreesh, TJ McKinley, J. Oakley, R. Nsubuga, M. Goldstein and R. White*



# Outline

- Motivation.
- Emulation.
- History matching.
- Results.

# Calibration of computer models



## Calibration objective:

To find a set of input values so that the simulator represents best the physical process as this is described by observations.

# The 'Mukwano' simulator

- A dynamic, stochastic, individual based model that simulates heterosexual sexual partnerships and HIV transmission.
- 22 inputs inc. contact rates, concurrency parameters, relationship duration, 2 sexual activity groups (high/low), 2 concurrency groups (high/low), 3 discrete behaviour periods.
- 18 outputs inc. population size, HIV prevalence, prevalence of men and women in long/short duration partnerships with one or more partners.
- Run time varies from 10 mins to >3 hours for 1 simulator run.
- Calibration data provided by a general population cohort in Uganda.

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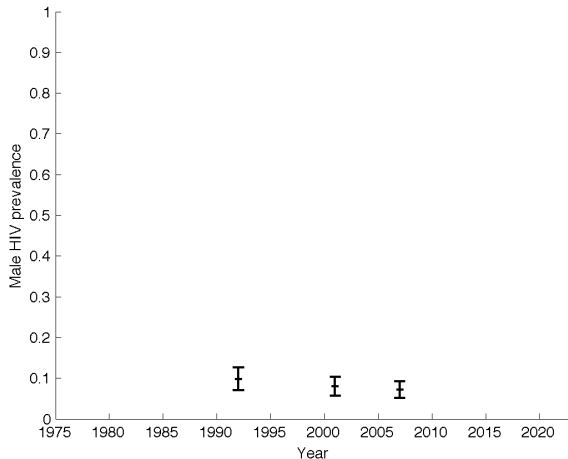
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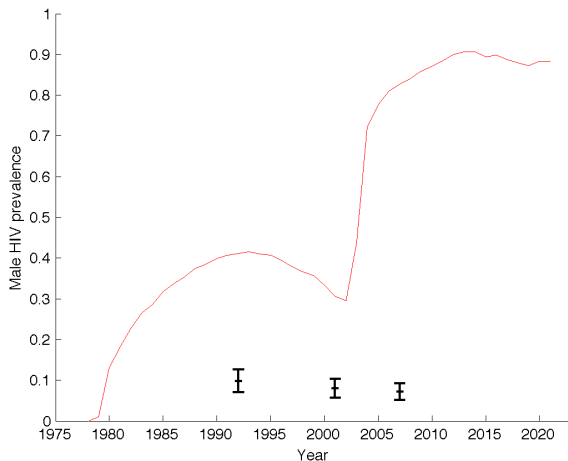
# A manual approach

Suppose we want to match male HIV prevalences at 3 points in time.



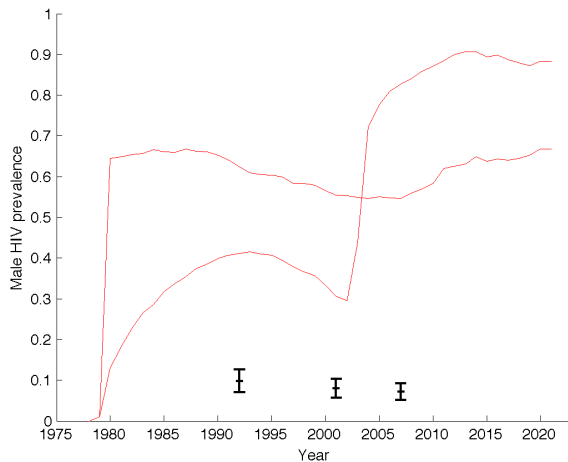
# A manual approach

We choose a set of inputs run the model and...



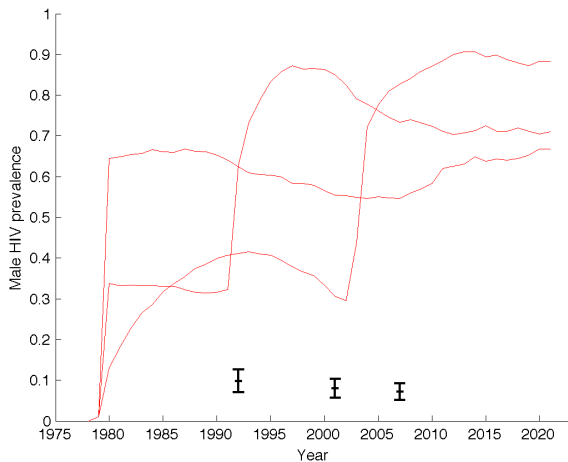
# A manual approach

...we try again...



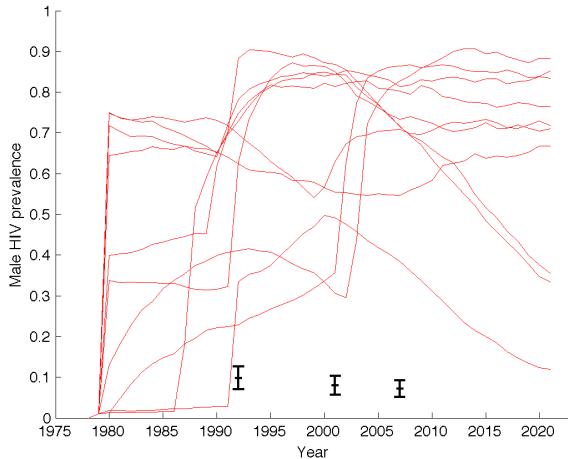
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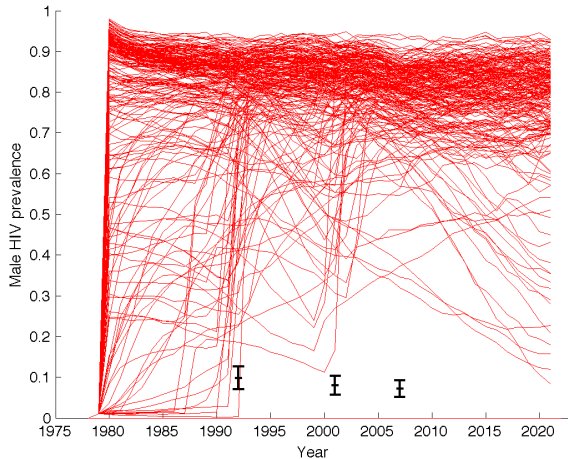
# A manual approach

...after 10 runs...



# A manual approach

...after 250 runs.



# History matching

- Rather than looking for the best input values, history matching identifies and discards those unlikely to provide a match to the empirical data.
- The *implausible* input space is discarded in iterations known as waves.
- Not all inputs/outputs need to be considered at once.
- The simulator is often ‘better behaved’ in smaller areas of input space.
- History matching relies on *emulators* for computational efficiency.

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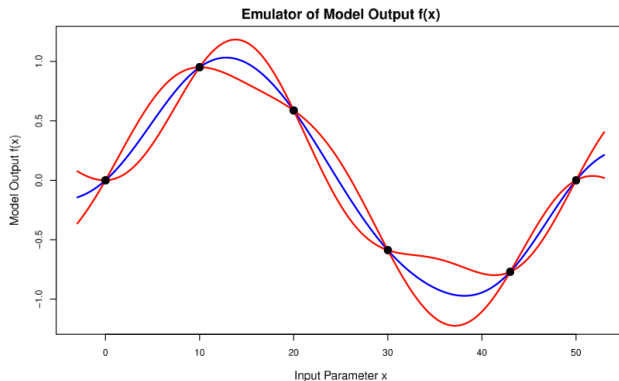
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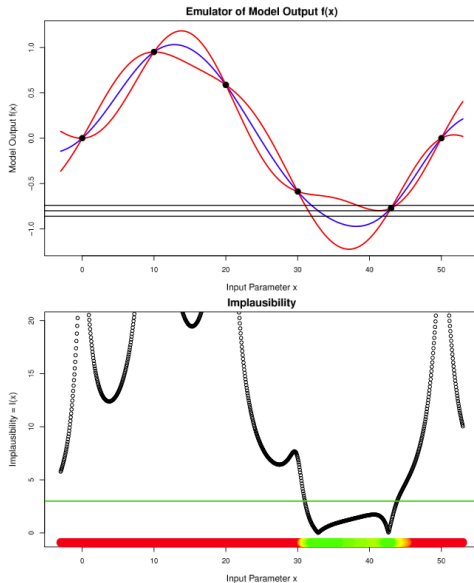
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# An emulator example

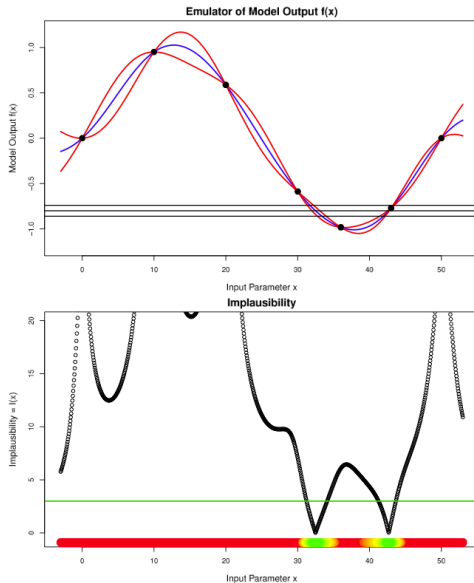
The emulator gives a posterior distribution for the model output, conditioned on the model runs we have seen so far.



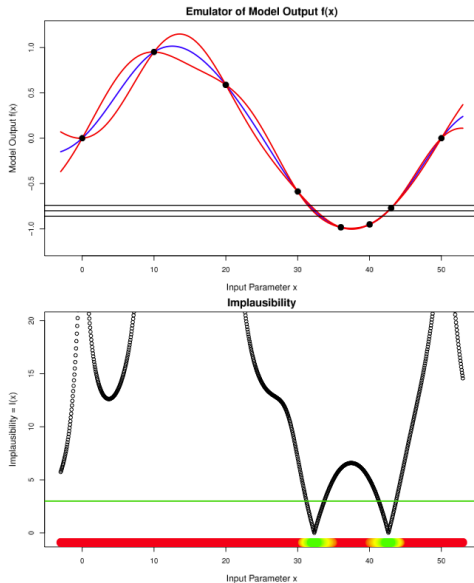
# History matching - wave 1



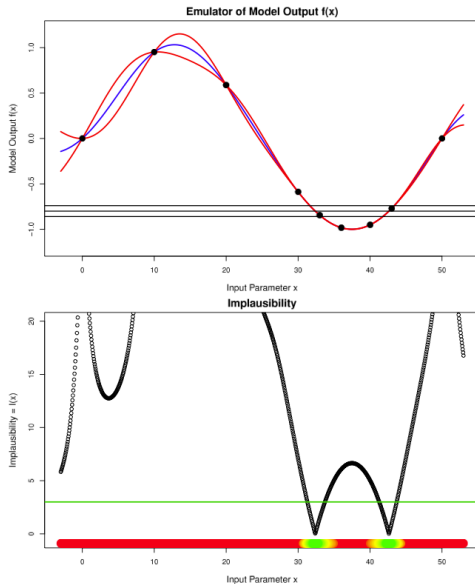
# History matching - wave 2



# History matching - wave 3



# History matching - wave 4





# Challenges

- Large number of inputs and outputs.
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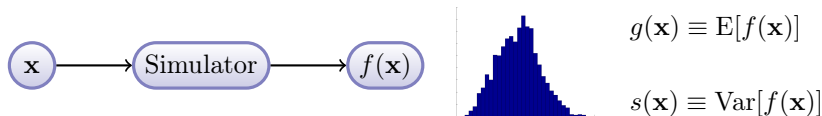
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For an input  $\mathbf{x}$ , the simulator's output is a draw from an unknown distribution, with mean  $g(\mathbf{x})$  and variance  $s(\mathbf{x})$ .



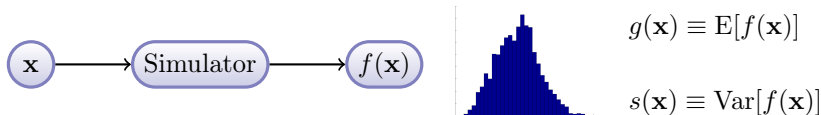
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$$f(\mathbf{x}) = g(\mathbf{x}) + \epsilon(\mathbf{x})$$

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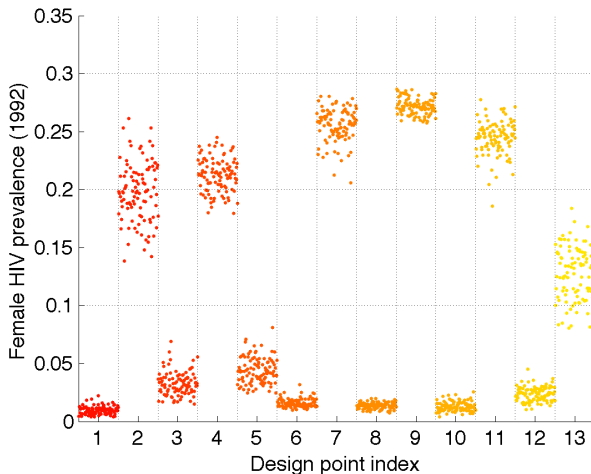
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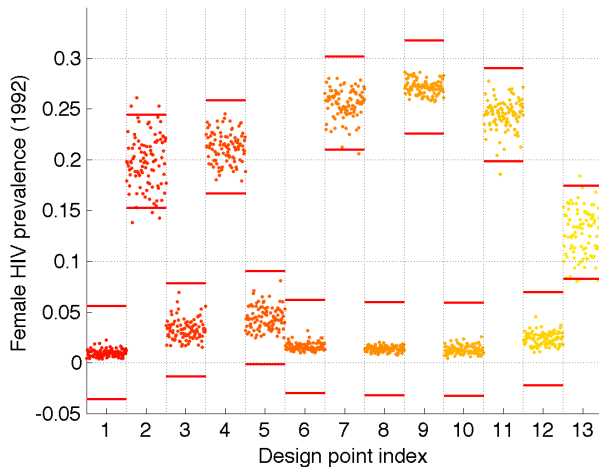
The variance  $s(\mathbf{x})$  is a function of  $\mathbf{x}$  and this has to be taken into account in history matching.





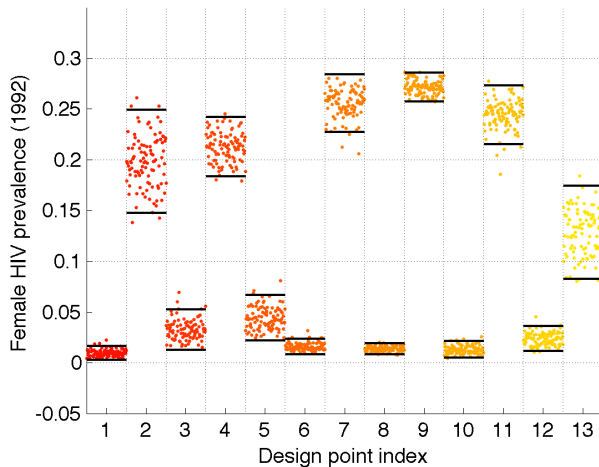
# The variance $s(\mathbf{x})$

One approach is to assume a fixed variance across  $\mathbf{x}$ .

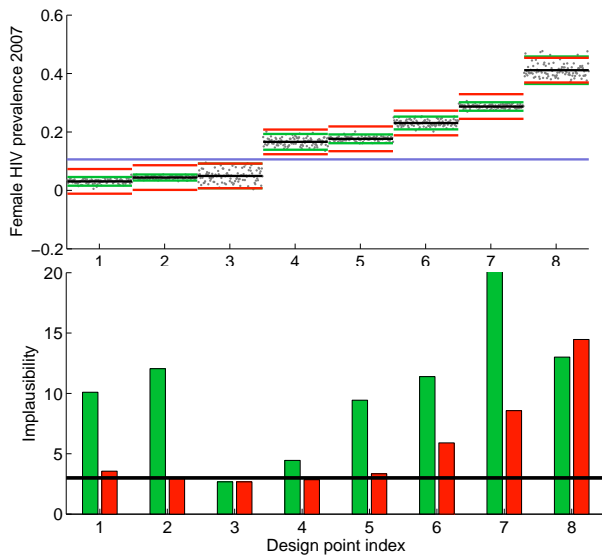


# The variance $s(\mathbf{x})$

Estimating the variance can improve the results.



# An example



- Evaluate  $f(\mathbf{x})$   $K$  times at each of  $N$  different inputs  $\mathbf{x}_n$ .
- Calculate means and log-variances

$$\hat{g}(\mathbf{x}_n) = \frac{1}{K} \sum_{k=1}^K f_k(\mathbf{x}_n), \quad \hat{\xi}(\mathbf{x}_n) = \ln \left( \frac{1}{K-1} \sum_{k=1}^K (f_k(\mathbf{x}_n) - \hat{g}(\mathbf{x}_n))^2 \right).$$

- Gather training data  $D = \{\mathbf{x}_n, \hat{g}(\mathbf{x}_n)\}$  and  $D' = \{\mathbf{x}_n, \hat{\xi}(\mathbf{x}_n)\}$ .
- Use a GP prior on  $g$  and  $\xi$ , i.e.  $g(\mathbf{x}) \sim \mathcal{GP}(\cdot, \cdot)$ ,  $\xi(\mathbf{x}) \sim \mathcal{GP}(\cdot, \cdot)$ .
- Calculate posteriors:  $E^*[g(\mathbf{x})]$ ,  $\text{Var}^*[g(\mathbf{x})]$ ,  $E^*[\xi(\mathbf{x})]$  and  $\text{Var}^*[\xi(\mathbf{x})]$ .

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# Some emulator details

- $g(\mathbf{x}), \xi(\mathbf{x}) \sim \mathcal{GP}(h(\mathbf{x})\beta, \sigma^2(c(\mathbf{x}, \mathbf{x}') + \nu))$
- $h(\mathbf{x}) = 1 + \mathbf{x} + \mathbf{x}^2 + \mathbf{x}^3$ .
- $c(\mathbf{x}, \mathbf{x}')$  is the Matérn 3/2 correlation function.
- $\beta, \sigma^2$  are marginalised with  $p(\beta, \sigma^2) \propto \sigma^{-2}$ .
- Correlation lengths  $\delta$  and nugget  $\nu$  are estimated with maximum likelihood.
- All outputs are emulated independently.

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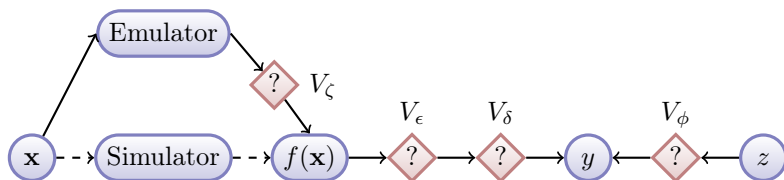
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# Uncertainty structure



$z$  : Observations

$y$  : Physical process

$f(\mathbf{x})$  : Simulator's output

$\mathbf{x}$  : Simulator's input

$V_\zeta$ : Code Uncertainty

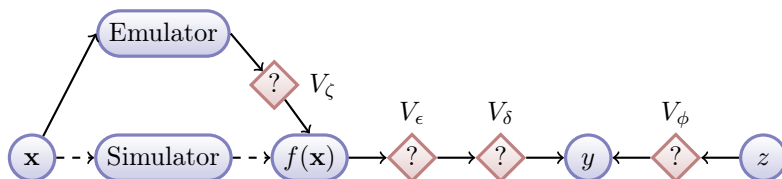
$V_\delta$ : Model Discrepancy

$V_\epsilon$ : Ensemble Variability

$V_\phi$ : Observation Uncertainty



# Uncertainty structure



- We link  $z$  with the posterior expectation of  $g(\mathbf{x})$ 's emulator via

$$z = E^*[g(\mathbf{x})] + \zeta + \epsilon + \delta + \phi$$

where  $\zeta, \epsilon, \delta, \phi$  are zero mean unimodal random variables.

- The variances  $V_\delta, V_\phi$  are provided by the model experts/data.
- $V_\zeta(\mathbf{x}) = \text{Var}^*[g(\mathbf{x})]$  and  $V_\epsilon(\mathbf{x}) = \exp(E^*[\xi(\mathbf{x})])$ .

# The implausibility measure

- The link function  $z = E^*[g(\mathbf{x})] + \zeta + \epsilon + \delta + \phi$  allows us to write the implausibility measure as:

$$I(\mathbf{x}) = \frac{|z - E^*[g(\mathbf{x})]|}{(V_\zeta(\mathbf{x}) + V_\epsilon(\mathbf{x}) + V_\delta + V_\phi)^{1/2}}$$

- A large value of  $I(\mathbf{x})$ , indicates that  $\mathbf{x}$  is unlikely to result in a good match between the model and the data.
- A small value of  $\mathbf{x}$  does not imply that  $\mathbf{x}$  is good! We do not know yet.
- The magnitude of  $I(\mathbf{x})$  is often judged based on Pukelsheim's  $3\sigma$  rule.
- The use of emulators allows to evaluate the implausibility almost instantaneously.

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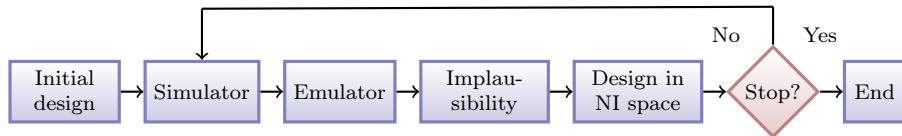
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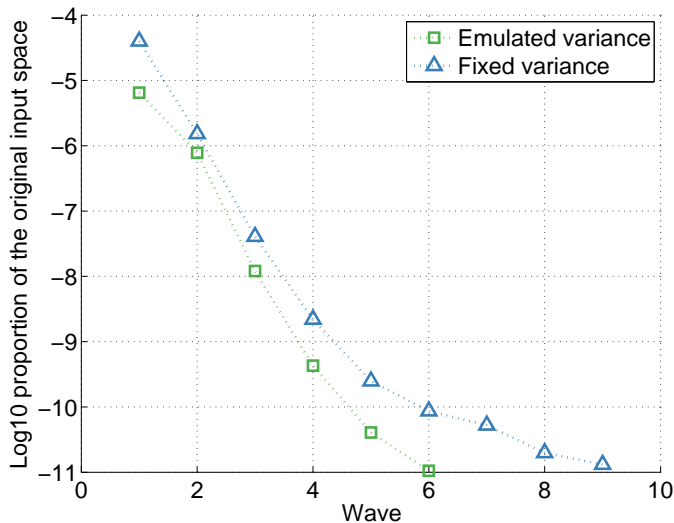


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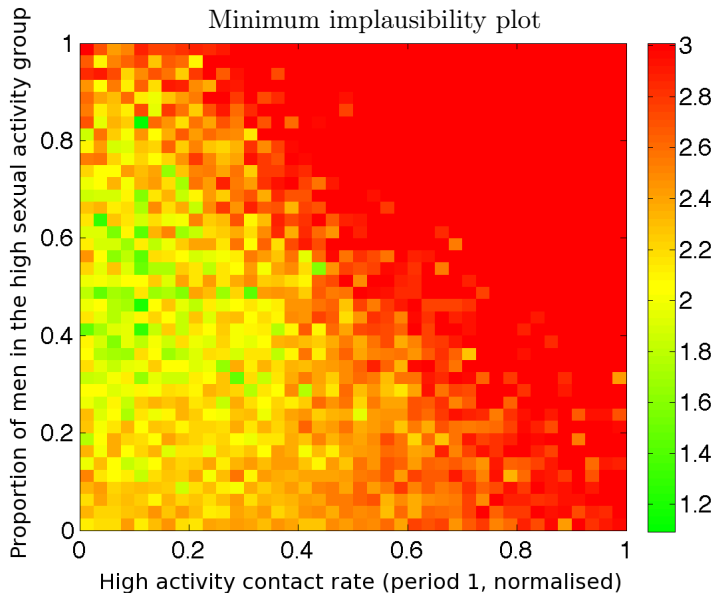
# Rejection rates



# Visualising the implausible space

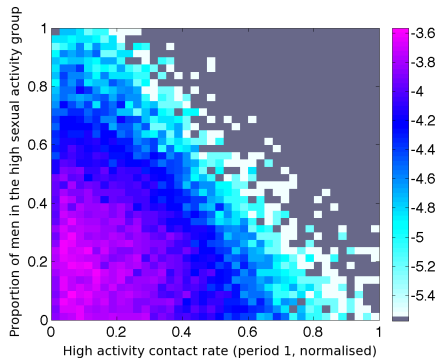
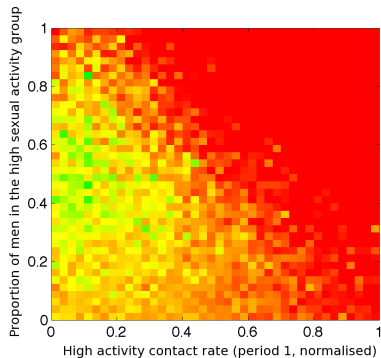
The implausible space can be visualised with minimum implausibility and optical depth plots.

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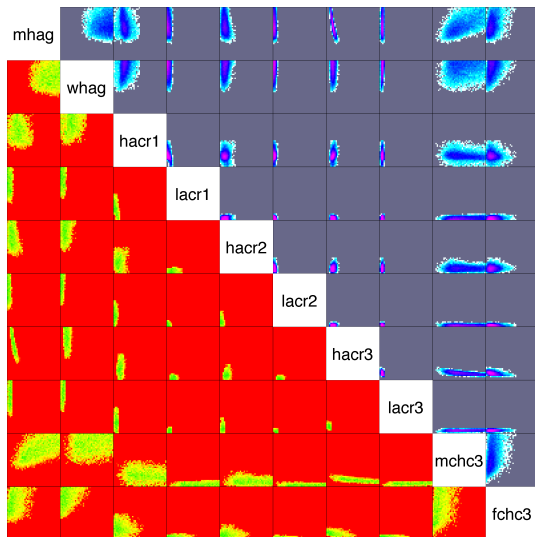


# Visualising the implausible space

## Optical depth plot

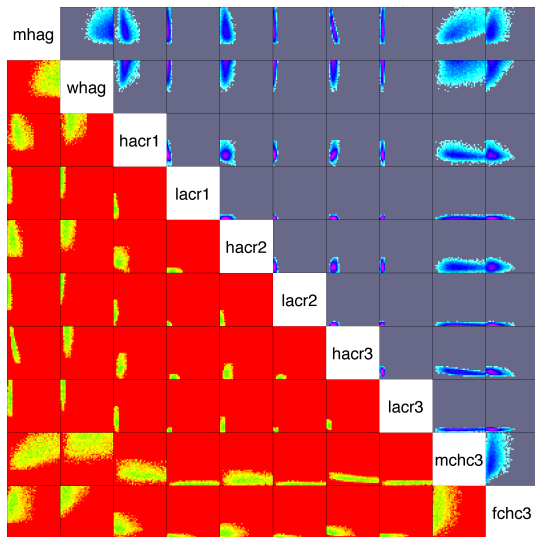


# Implausibility plots wave 9 (fixed variance)

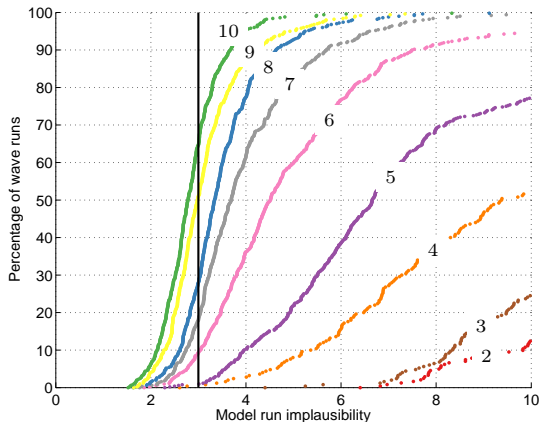




# Implausibility plots wave 6



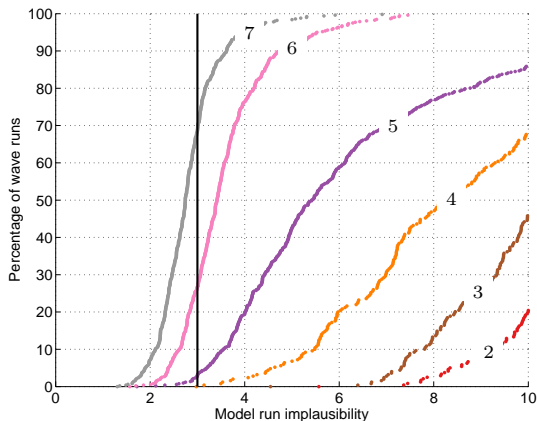
# Implausibility of model runs (fixed variance)



Implausibility of the *actual* simulator runs (no emulation involved)

$$I(\mathbf{x}) = \frac{|z - \hat{g}(\mathbf{x})|}{(\hat{s}(\mathbf{x}) + V_\delta + V_\phi)^{1/2}}$$

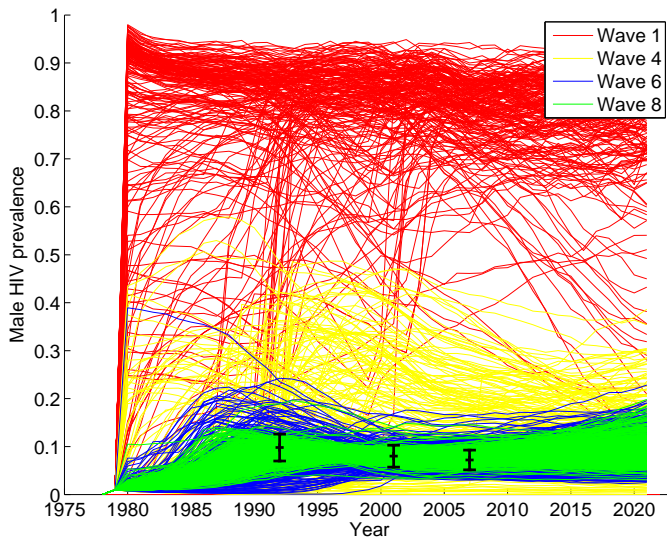
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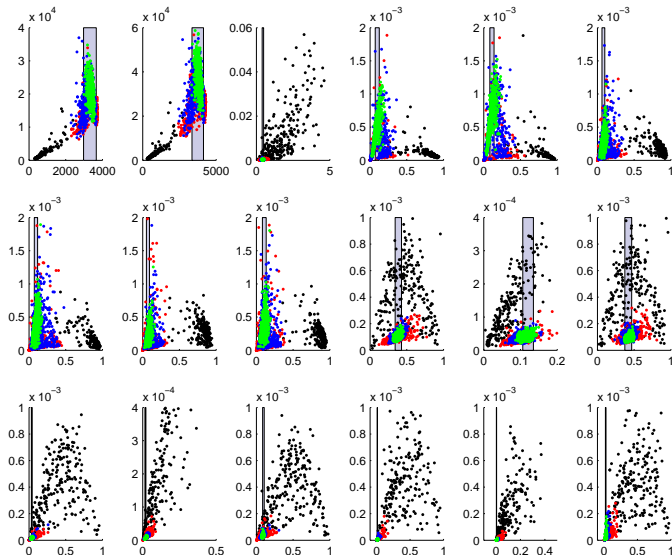
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# Output matching



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# Conclusion

- We extended history matching so that it can efficiently handle stochastic models.
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- Predicts the effects of ART on mortality and transmission over the next 15-20 years.
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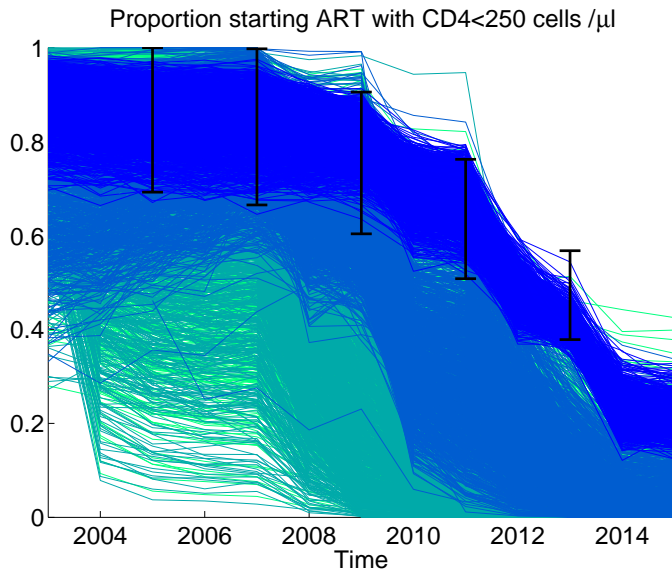
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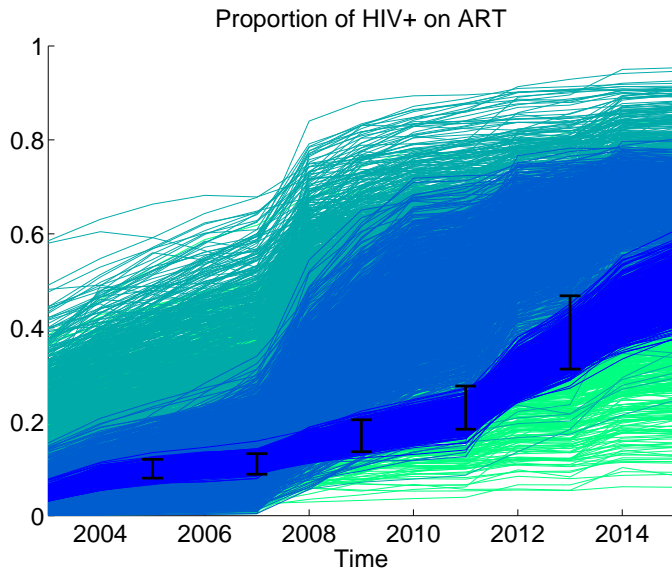
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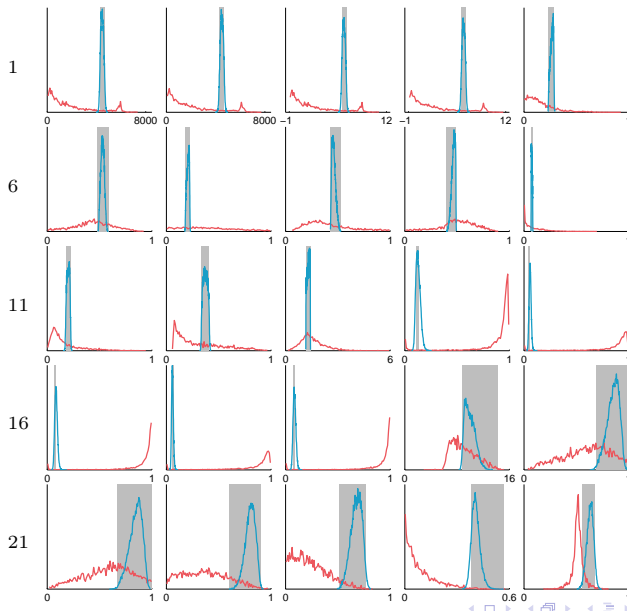
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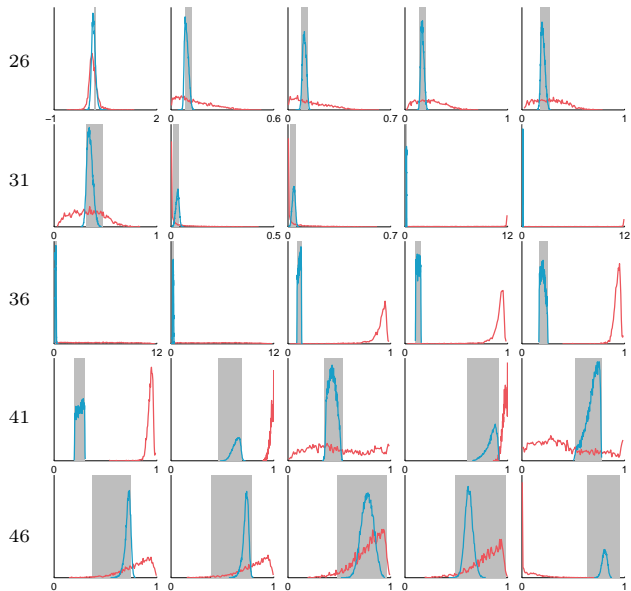
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- History matching provided hundreds of input points that match all the outputs simultaneously.
- These inputs are used to run the simulator into the future and predict the effect of different ART interventions to mortality, HIV prevalence etc.
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